



Be a Mathemagician!

By Andrew Herrmann, Department of Mathematics

aherrman@sfsu.edu



SAN FRANCISCO STATE UNIVERSITY

Supported by San Francisco State University's (CM)² program, a branch of the NSF GK-12 Program (Grant DGE-0841164)

Summary

Borrowing from Tom Davis and Keith Riding, I demonstrated a card trick to Bayview Math Circle students, and subsequently allowed them to explore how it works. As in a typical card trick, the subject chooses a card, and I inevitably tell them the card. As a twist, the subject is allowed to determine where in the deck the card will appear (eg fifth card, 23rd card, etc.). After presenting the trick several times, students will be encouraged to discover the mechanics of the trick. The trick hinges on use of modular or base three arithmetic. Goals include developing inductive reasoning skills, learning base 3 arithmetic and simple combinatorics.

Prerequisites

For students

- Basic combinatorics (not required)
- Reasoning skills
- Age group: any



For instructors

- Some modular arithmetic knowledge
- Basic combinatorics

Significance This activity gives an excellent introduction to modular arithmetic and combinatorics, both of which have a variety of applications. Below is a short list.

Modular Arithmetic Applications

- Error correction in credit cards, finance and science
- Cryptography
- Music
- Binary arithmetic
- Social sciences
- Law



Combinatorics

- Computer science
- Optimization
 - Shipping
 - Passengers boarding planes
 - Electrical wiring
- Probability
 - Casino games
 - Error correction/data verification
 - Genetics



Simple Version

Our goal is to be able to determine a card chosen by a student using information about the location of the card within one of three columns.

Execution

1. Choose 27 cards from a standard deck. Choosing specific cards and ordering them will help students understand
2. Deal the cards out in three columns of nine
3. Have the student choose one card, not revealing their choice
4. Have the student reveal the column the card is located in
5. Pick up the columns, preserving order within columns
6. Deal cards back out, from left to right
7. Repeat steps 3-6
8. Ask the student to reveal the column the card is located in
9. Reveal the card to the student

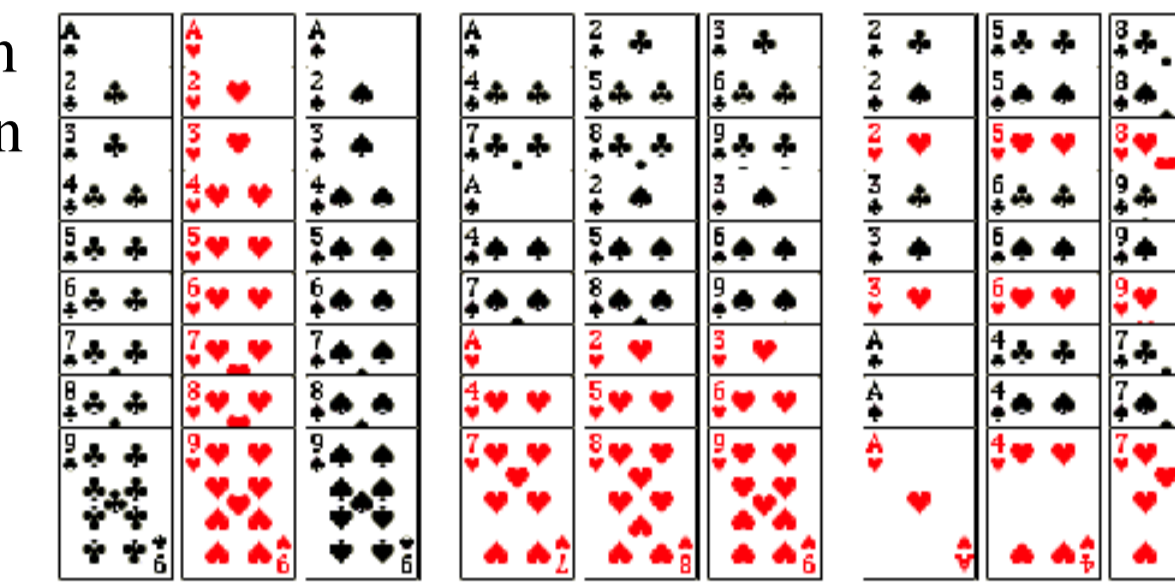


Figure 1: Organizing cards aids in understanding

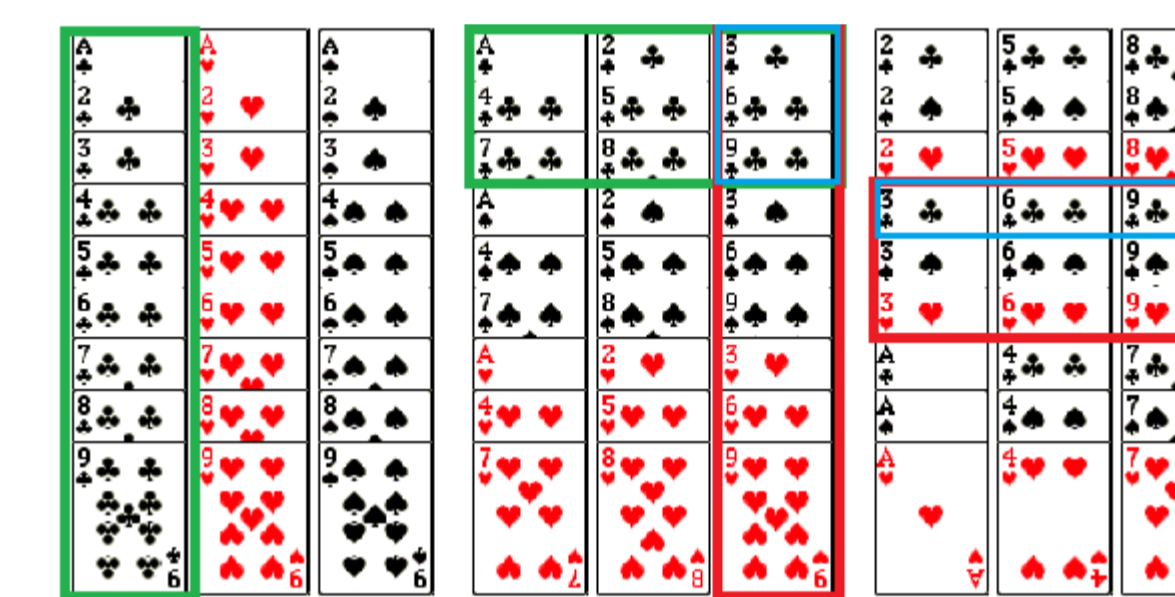


Figure 2: The choice of columns will reveal the card



This is easy to see using our ordered cards. The first time a student chooses a column, we know the suit of their card. The second time, we know the cards equivalence modulo 3. When we deal the cards out a third time, there are only three options for the student's card. When they tell you the column this time, we know their card.

As stated before, this gets trickier when cards are unordered—it's hard to recall which cards where in each column initially, and no properties are revealed by the card's location within a column.

Simple Ordering

The example below can be done using arbitrary cards; we can also replace first with second or third, with analogous results.

- Perform the trick as usual
- Pick up the chosen column first each time
- Maintain order: deal out the first column first
- Pick up cards after the column is pointed out the third time
- The top card is the student's card

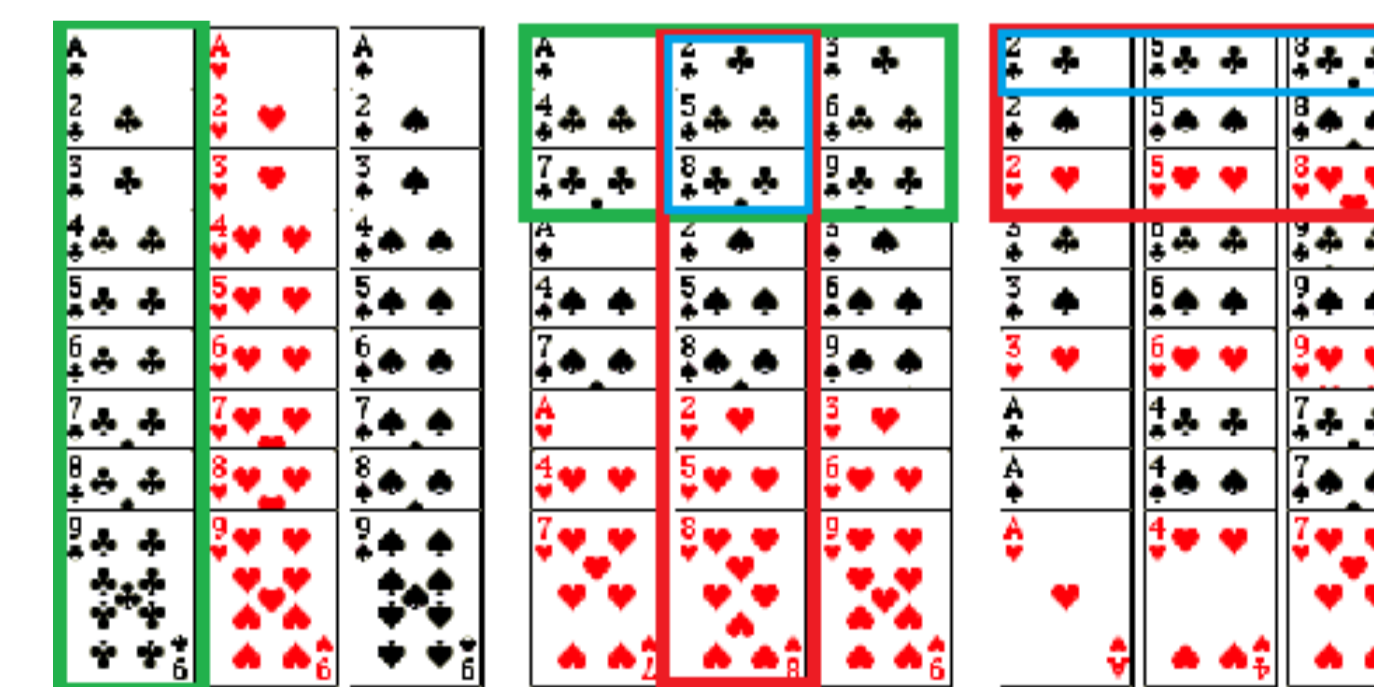


Figure 3: Picking up the students column first each time. The student's card is moved up by one-third on each pickup.

Common Problems

- Students had trouble discovering how the trick worked
- Students had little combinatorics knowledge, missing why the trick worked

To help students struggling with the mechanics of the trick, we can simplify the trick by using a smaller selection of cards. For example, we can give the students a two-by-two array of cards, and repeat the trick (where they choose the column twice rather than thrice). In this case, students were quicker to discover how the trick worked, though they still required some guidance when asked to explain the larger version.



Recommendations

- Provide students with instructions on how to perform the trick
- Give students simple combinatorics problems as a warmup
- Give students a modular arithmetic lesson (can be part of a math circle activity or an actual lesson)



Final Thoughts

The primary difficulty for most students was not figuring out how the trick worked—most students understood the mechanics of the trick fairly quickly, and could replicate it at will. Rather, many students struggled with *why* it worked. Few seemed to understand that the number of cards, size and number of columns, and number of pickups were all very closely related. However, in this math circle session, we presented a much more complicated version of the same trick.